

STEP I 2002 Comments

Question 1

A nice first question on the paper here. The first integral uses a standard trigonometric identity and the second falls out after using the given substitution. For the third integral, the form of the limits should suggest using a substitution of the form $y = \frac{1}{x}$. After doing this, it turns out to be identical to the second integral.

Question 2

A nice technique, although it can only be applied to equations of a rather particular form! The second part of the question is very well designed to yield to the technique twice in a row. It does lead to a fair bit of repetitive work but I forgive it for the well-chosen polynomial!

Question 3

This is a nice question. The induction proves a nice result about Fermat numbers which is then used as an alternative proof of the infinitude of the primes. As it turns out F_0, F_1, F_2, F_3 , and F_4 are all prime, and the only known Fermat primes!

Question 4

A nice question here. Not too much algebra to work through and some thought is needed to choose the right p and q for (i) and (ii) and the right function for (iii). The form of the required answer in (iii) should be suggestive of the integrated version of the function to choose.

Question 5

The recurrence $x_{n+1} = kx_n(1 - x_n)$ is the *logistic map*, a dynamical system noted for its chaotic behaviour. It is nice here to find its fixed point as well as cycles of length 2.

Question 6

I think this one is rather tricky! Interpreting the information in the question into a useful diagram is of course the place to start but even drawing the 3D diagram in a way which is helpful is not trivial. Once the first part is done I think the second part is actually not that bad; it's the same starting point as before just with a trig equation to solve halfway through.

Question 7

Another tricky one, with some tricky 3D visualising to do again! None of the algebra is particularly horrible but it does take some getting through.

Question 8

This seems much easier than the previous two questions! I don't think there are any major stumbling blocks here, just working through the maths to get the results.

Question 9

This is quite tricky, I think. There's a fair bit of algebra to work through, and the second part is a little different to what is normally asked in these projectile questions.

Question 10

I found the first part of this question more difficult than the second, getting confused between t s and T s and the slightly fiddly differentiation! The maths is not too difficult but being checking the various cases and get everything accurate takes some care.

Question 11

A pretty standard question here, I think. The standard statics methods work fine to get the result, and then it's just a case of substituting in the given values to find μ .

Question 12

I'm not really sure what to make of this question – I think it's a little fiddly and there's not much mathematical interest to it, to be honest.

Question 13

Another slightly odd question. The first part is quite nice, showing that $G(y)$ is indeed a well-defined cumulative distribution function. The rest follows from some integration (and carefully making sure things are positive where required).

Question 14

A much more solid question! Some confident manipulation of series required throughout, both polynomials and ones with exponentials. Nothing too terrible here, I think.