

STEP I 2001 Comments

Question 1

A decent enough first question, but I always dislike these sorts of questions on a non-calculator paper. The arithmetic is not particularly difficult, but annoying!

Question 2

A reasonably straightforward question, once you have figured out what the $\left[\frac{k}{N}\right]$ function is doing. The algebra is a little fiddly but not particularly long.

Question 3

A nice little question, with some need to tread carefully when justifying the modulus and then when explaining where functions are increasing or decreasing.

Question 4

A little fiddly with the algebra. Using the hint in the first part should suggest also using the double angle formulae at which point you can factorise $\sin x$ and leave a quadratic in $\cos x$. For the second part it seems natural to proceed as previously, this time leaving a factorisable quadratic. You then just need analyse the roots of each factor.

Question 5

The differentiation and integration here is fairly straightforward. The sketch requires considering the behaviour of the curve as $x \rightarrow \pm\infty$. The final part proving the inequality on k is a little tricky.

Question 6

Fairly tricky, I think. There are a number of ways to approach the first integral – but recognising the fraction is top-heavy and dealing with it appropriately is sensible. It took me a while to spot you could do the second part by parts; originally I did a substitution which led to the third part! It helps that the derivative of $\arctan \frac{1-x}{1+x}$ is so neat.

Question 7

A little messy with the algebra but turned into a nice result in the end. Lots of careful work with square roots here! It is important to justify if e.g. $x^2 = 3$ why $x = \sqrt{3}$ and not $-\sqrt{3}$.

Question 8

Quite a short and straightforward question, I think. Maybe it would be better to do (ii) formally using induction but I think the result is clear enough.

Question 9

The tricky thing here is dealing correctly with all of the different constant and variables and knowing when to write different things in terms of others. The integration is all straightforward but knowing what to do when is harder.

Question 10

This is a fairly standard 1D collisions question. The final part is a little bit more involved with the algebra. Subtracting $\frac{mu^2}{27}$ from the expression for energy and showing it is positive is easier than trying to show the expression itself is bigger than $\frac{mu^2}{27}$, which would involve a fairly messy completing the square or differentiation and substitution.

Question 11

A couple of applications of suvat is what is required here, followed by some rearranging. There is a little bit of care required to check whether things are positive or negative when rearranging the inequality.

Question 12

I think this is fairly tricky. Dealing with binomial probabilities is always bit fiddly and it is easy to make a mistake on the first part to find the maximal probability. The second part is also hard, requiring the use of the Poisson approximation to the binomial and then also approximating the finite series with an infinite series.

Question 13

I'm not a massive fan of this question. Finding the approximate value of α seems a little messy and I don't like using m and M . Much better to use μ for the mean to avoid confusion.

Question 13

A nice little question involving a weighted average sum of two Bernoulli random variables. Finding the mean is easy, and finding an expression for the variance is not difficult. After that you must differentiate to find the minimum variance and then justify it is a minimum. The final big step is then finding a simplified expression for this minimum variance. To be honest, I think the final part involving the normal distribution is a little unnecessary!