STEP I 2000 Comments

Question 1

The geometry and algebra here is not tricky – I think the hardest bit is justifying that an equilateral triangle will give the maximal length of the shortest side.

Question 2

A straightforward inequality question here, the difficulty of course dealing with the asymptote in the first part and fictitious solutions in the second. Both of these can be resolved with a good sketch.

Question 3

A nice way to prove the result at the start using stationary points. After that, it is important to understand what something being a necessary or sufficient condition is. If something is necessary, if it is not true then the result is not true. If something is sufficient, then if it is true, then the result is true. We say a condition is "necessary and sufficient" if it is logically identical to the result. For example, the discriminant of a quadratic expression being positive is necessary and sufficient for the expression having two distinct real roots.

Question 4

A little fiddly this one, requiring good understanding of the domain and range of the inverse trigonometric functions.

Question 5

This is a special case of the Leibniz rule for differentiating under the integral sign, a useful technique when you have a function of two variables being integrated with respect to only one of those. All the integration in the question is straightforward and I do not think should pose much of a problem.

Question 6

Quite a neat question, and a nice result about how the surface area remaining is proportional to the number of slices remaining.

Question 7

A reasonably doable question, I think. Solving the differential equations should not be difficult, but reasoning about the possible values of a and b requires some thought.

Question 8

Another question on differential equations. This begins fairly gently, but the second part is a little trickier. It becomes much easier if you spot that (as you just showed) any linear combination of the two solutions is also a solution, and choosing these judiciously allows you to find p and q quickly.

Question 9

This is a nice question. A good diagram is of course vital! The key insight here is finding some distance (or time) that you want to minimise as a function of θ, and then differentiating to do this.

Question 10

I am impressed at how many varieties of these sort of projectiles questions they can come up with! Here there seem to be two degrees of freedom with the projection – both the horizontal and vertical angle – but subject to the constraint that the shell must hit the railway line, these reduce to one constraint. So eliminating either of these angles and then maximising flight time with respect to the other one is the way to go.

Question 11

Quite a straightforward question on elastic strings here – once the diagram is correctly drawn, the rest is just working through the algebra from the standard equations for energy and tension in an elastic string.

Question 12

The difficulty here is getting the tree diagram correct. At each stage you need to look back and see who the current person does and doesn't know if they know the rumour. Once the diagram is correct, the probabilities are straightforward.

Question 13

Using the result that the sum of two Poisson variables is still Poisson is important here. Once that is used, it takes a little simplification to get the required result.

Question 14

A reasonably straightforward counting question. When listing the possibilities, it's quicker to just list the 12 possible first two candidates, rather than all 24 possibilities for all four candidates.