STEP II 2000 Comments

Question 1

It's easy to get going in this question, finding the equation of the relevant tangent and normal, but the algebra turns out to be a fair bit trickier than expected. Because of the two equations linking a and b, there are many ways to write each answer and you need to use both equations to simplify to get $-b$ or a.

Question 2

This is a nice integration question. You can start the first part with the given substitution but then need a little ingenuity to simplify the integrand. The second part follow naturally, with the form of substitution hopefully suggesting itself, with then some work to do to establish exactly what substitution to use.

Question 3

There's a lot going on here and so a large, clear diagram is of course vital! Being able to rotate a point in an Argand diagram about any other point is an important skill done by translating by the centre of rotation, rotating, and then translating back.

Question 4

A nice question, ending with a fairly complicated graph to sketch. The first part relies on keeping track of x, c, and a, and which are constant and which are variable at each stage. Working through that gets the first result. Finding the oblique asymptotes of a rational function can be done by long division. Sketching the graphs then becomes a process of drawing the asymptotes, and then using where the graph is positive, negative, and zero to find its shape. Putting this equation into Desmos and gradually changing the value of a gives a nice visualisation.

Question 5

This is a tough one, I think. $\Delta(\boldsymbol{a}, \boldsymbol{b})$ is the determinant of the matrix with columns $\boldsymbol{a}$ and $\boldsymbol{b}$ and so is the scale factor for areas under this transformation. I can't quite go from there to get the intuition for why the final result holds, though!

Question 6

A nice question. When I worked through this I was confused about why I was asked to verify the result in a particular case, which of course becomes clear later on! Spotting you can use a substitution to turn the final expression into one of the form studied earlier is helpful, but I don't think it would be too bad to do otherwise. The two factors are either $(x^2 + ax + 3)(x^2 + bx + 13)$ or $(x^2 + ax + 1)(x^2 + bx + 39)$ and you can work out which values of a and b work from there.

Question 7

Some nice analysis here, using the AM-GM inequality to prove quite a nice result. Working out a way to write the product in a form which the previous result can be used if of course the hard bit here. Playing around with the numbers will hopefully lead you to the right place.

Question 8

A fairly lengthy question. The first induction is on the tricky side as it requires using both the inductive hypothesis and also the original definition. The next part involves using the standard method for solving linear difference equations (analogous to linear differential equations). If this hasn't been seen before, this will be tricky.

Question 9

A fairly abstract, but short, collisions question. Dealing with things in two dimensions always makes things a little more tricky! I think the hard part is writing down the write equations, as after that the algebra is reasonably straightforward.

Question 10

A fairly standard (but not straightforward!) statics question. As ever, a large, clear diagram is vital, showing all of the relevant forces. Then being sensible when choosing which equations to write down (and writing them down correctly!) will get you through.

Question 11

I think this is pretty tough, requiring strong knowledge and understanding of rotational motion at a variable angular speed. The algebra isn't too hard, but writing down the correct equations is!

Question 12

Writing down the first expression shouldn't be too tricky, but then it requires a fair bit of manipulation to get it onto a form where the limit expression for e^{-x} can be used. I think showing that this probability is approximately $\frac{2}{3}$ is a bit of a waste of time!

Question 13

A reasonably long question, with a fair bit of involved algebra, particularly when dealing with the infinite series in the second part. Probability generating functions (and their cousin for continuous random variables, moment generating functions) are incredibly useful tools to understand the properties of a distribution.

Question 14

Independence (or not) of random variables is an important topic, and in general $E(XY) = E(X)E(Y)$ is necessary, but not sufficient, for independence. The final part of this question is a good example of how this is not sufficient. In general, we say two variables are independent if their joint probability mass (or density) function $f(x, y)$ can be factorised as $g(x)h(y)$.