STEP I 2000 Comments

Question 1

I usually don't approve of numerical questions, but I do quite like this one – probably because no more arithmetic is needed than adding, subtracting, and multiplying by two or powers of ten! I like the ramp up in difficulty (a very handy decimal expansion of $\log_{10} 2$!) and working out the first digit of very large numbers like that is, I think, an interesting problem.

Question 2

This is one of my favourite questions. The skills required to answer it are learnt in the first year of an A Level course, but to get through the question without a lot of very dense and messy algebra requires some thought and ingenuity.

Question 3

A fairly short question. I think the main difficulty comes from getting the correct shape of the graph of $y = [e^x]$. It does require a bit of thought to work out how long each of the pieces is. Once you have the correct sketch, establishing the final result shouldn't be too tricky. The function in this question is called the "floor function" and is often denoted by $\lfloor x \rfloor$.

Question 4

A lot of algebra, here! I think a little too much, as it takes away from the focus of the question which is about bounding functions and hence their integrals. The actual value of the integral is $\frac{11}{48} + \frac{5\pi}{64} \approx 0.47460$. This can be done via some messy partial fractions and an arctan integral.

Question 5

I think this is a bit tricky. Drawing a good diagram and getting a sense in your head of the geometry of the problem is of course essential but also non-trivial.

Question 6

Some inequalities. It takes a bit of work to get the brackets right for the second part – the basic structure should be fairly clear but some fine tuning is required. I think the final part would be more interesting if there weren't any points which satisfied both inequalities – it would require a better argument than just giving a point that does satisfy both.

Question 7

I think the difficulty here comes from maintaining the logical flow of the argument. It is easy to reverse the direction of the implication at some point, or not making clear why your final line does indeed prove the required result. The actual algebra is fairly straightforward.

Question 8

I'm not exactly sure what the first "show that" wants you to write down – the statement itself seems obvious! I think this is a fairly easy question, with the integration itself being trivial, the only difficulty coming from working out where to split each integral – but even that isn't very hard. Using the symmetry of the sine curve can reduce the work you need to do for the last part.

Question 9

A little more difficult than a “standard” STEP I projectiles question, I think, having to deal with the acceleration of the train as well as the trigonometry, and having to decide what approximations can be justified.

Question 10

When I was preparing for STEP, I always liked these 1D collision questions because the method was usually the same – set up conservation of momentum and restitution equations for each collision, and then solve them simultaneously. That’s pretty much what you need to do here – with a little bit more at the end to find the range of possible final energies.

Question 11

This is a nice statics question, testing that the candidate understands that friction is not always limiting! As always with these sorts of questions, a clear diagram is essential, as is clear statement of the various equations and where they come from.

Question 12

A fairly simple probability question, I think. Most of the difficulty comes from correctly parsing the information in each part of the question – once you understand that, the rest shouldn’t be too bad.

Question 13

Some conditional probability here. Knowing and using the definition of conditional probability is of course key to questions like this.

Question 14

Knowing the properties of expectation is important here – in particular that $E(X+Y) = E(X) + E(Y)$ for all random variables, and that $E(XY) = E(X)E(Y)$ if X and Y are independent. The “show that” at the end is a nice way to check you’ve got the question correct!