STEP III 1999 Comments

Question 1

Roots of polynomials is a topic that I don't really like in the further maths specification. There's some nice symmetries in there but I personally find it a little to algebra heavy. I feel a similar way about this question – there's a lot of algebra with lots of variables and constants in. A lot of the difficulty comes from working out exactly what variables or constants you're trying to eliminate.

Question 2

I like this question, but it does feel a bit more like STEP II than STEP III material. The key, of course, is working out the length between the two parts of the question, and how to exploit this, and understanding why the answer will be different for the two ranges of values of k.

Question 3

This is a nice question. The fairly scary-looking equality at the start is not so difficult once you realise the left hand side is just the sum of the areas of various rectangles. There's a nice progression to the two parts.

Question 4

This is a nice question, and quite different in flavour to most other STEP questions. The first part involves thinking about how the number of edges and vertices on the polygons that make up the polyhedron relate to the number of edges, faces, and vertices of the polyhedron itself. Proving the existence of only five regular polyhedra is an interesting exercise, and can be done by a number of different ways.

Question 5

Another induction question about Fibonacci numbers! They have so many nice properties. This is a neat one; I like the sort of double induction going on. This adds a layer of complexity and means you need to be confident knowing what you have proved, and what you have only assumed.

Question 6

Some polar coordinates. Hopefully, the parameterization is an obvious choice, with a little care to get the powers correct. Thinking about the sketch as a transformation of a circle is helpful. The final integration is entirely standard.

Question 7

A question on groups, which is no longer on the specification. It's a pretty decent question, and not too hard if you are confident with the group axioms.

Question 8

A fairly neat question on differential equations. I don't think it's that difficult – just needing to spot how to match up the boundary conditions each time. You could do a formal proof by induction, but I don't think it's necessary – the question says "find", not "prove". The sum $\sum_{n=1}^{\infty}\frac{1}{n^2}=\frac{\pi^2}{6}$ in fact, so I think giving this result and instead having to show that $\int_0^{\infty} y^2\,\mathrm{d}x=\frac{\pi^3}{6}$ might have been nice.

Question 9

I think this is really hard. It ends up with a very neat result, but I think it's not obvious how to do it (well it should be clear you need to integrate, but it's hard to get all the terms correct), and then there's a lot of messy algebra to work through, which is not obvious it's correct and everything cancels out until right at the end.

Question 10

Well, this was a nice change after the previous question! This feels too easy for STEP III, there isn't really any "problem" element – you just do what it says at each step. There are a couple of different ways to approximate the sum with an integral, but they should all lead to the same asymptotic result

Question 11

As is often the case with this sort of question, the difficulty comes from setting up the correct equations. Once that is done, the actual algebra is very easy.

Question 12

I think the hardest part of this question is the first part – working out how to evaluate the sum by recognising them as (almost) binomial coefficients takes a bit of spotting. Apart from that, as long as there is confidence rearranging and reindexing sums, the rest is pretty doable.

Question 13

Another of those questions where having a solid understanding of continuous random variables is important. The integrations are all trivial, so it's just a case of being able to set them up correctly (and knowing how to work out A) at the start.

Question 14

This is a tough question. If you haven't seen the trick in the first part (writing the summand as a derivative) I think it's hard to come up with that. For the second part, you need to be very accurate with lots of dense algebra, including reindexing sums, and using the sum of probabilities for a Poisson being one.