STEP I 1999 Comments

Question 1

A pretty doable number theory question to start. Using the inclusion-exclusion principle to find the number of integers is not too difficult, but finding their mean is a little more subtle. The second part is only milder harder than the first part, needing to note that $\frac{4179}{2}$ is halfway between multiples of 3, 7, and 21.

Question 2

This question is full of algebra! The x and y parts of the equation are identical so its quicker just to consider one of them and replace the other at the end.

Question 3

A nice question. None of the individual parts are particularly tricky, but it's possible to now know where to start, for example, (i). The final result is neat too, the value of x_1 being ϕ, the golden ratio.

Question 4

Hopefully, sketching (i) and then (ii) makes you think that graph transformations are a sensible way to approach this. It's not required, but does make the thinking a lot easier. Once you have an inequality with just $|x|$ and $|y|$ in it, you can consider the graph in the first quadrant, then just reflect it across the coordinate axes. There's a nice difficulty curve here, too, ending with a region defined by curves rather than straight lines.

Question 5

A good diagram is obviously key to getting into this question! The approximations are rather hand-wavy, as they generally are at this level, but having the confidence to use them is needed! I found the second part a little harder to follow, because it's ingrained in my head that in these sorts of problems capital letters represent large quantities and lower case letters represent small quantities, which is not the case here!

Question 6

Quite short and pretty doable, I think. Missing the case $c = 0$ in part (ii) is easily done. Apart from that, careful consideration of the different possible shapes of the quadratic graph is what is required.

Question 7

Most of this question is pretty standard – substituting the given functions into the equation and working through the algebra. The bit at the end is a little harder, I think, with some creativity needed to get the required result out.

Question 8

This is a really nice question and leads to a nice result, showing just how incredibly slowly the harmonic series grows, even though its sum to infinity is unbounded. A little care is needed to remember to show that $f(t) = \frac{t}{n(n-t)}$ satisfies the conditions given at the start of the question.

Question 9

The difficulty here comes from parsing the information in the question and setting up the equations – solving them is pretty straightforward. Also, I don't know who decided that using v and V in the same question was a good idea – these are really hard to distinguish when handwritten! Surely U and V would have been better?

Question 10

Some springs. As always, you'll need to understand the link between modulus of elasticity and the force exerted by the spring. Once you've written down the correct equation, the rest is fairly straightforward.

Question 11

A bit of geometry needed here, as well as a good understanding of circular motion. Once the force on each star is established it's not too tricky to rearrange to find the angular velocity. The second part is not much more tricky, just needing an extra force to be added.

Question 12

This feels a little hard and abstract for STEP I, I think. Nevertheless, both of the results are nice, once you get there. The inequality proved right at the start should of course set off your spider sense that it will be needed somewhere else in the question!

Question 13

Easy, when you spot the trick! The difficulty, of course, is spotting that you can use an (almost) binomial distribution to model the number of units. If you can't figure that out, then it's a very hard question!

Question 14

This one is fairly short and approachable. The first part is just using the definition of the expectation of a continuous random variable. The second part does require a little more thought, thinking about the different ways it's possible to win the game.