STEP II 1998 Comments

Question 1

I'm not really sure what the point of this question is. The different parts feel quite distinct from each other, and I don't think the final result is particularly interesting.

Question 2

This question is about the beta function, and important function in complex analysis, which is just a slightly shifted version of I. This is a nice question exploiting various properties of the integral to eventually find a closed from expression for its value (and in fact this result is valid for all complex numbers with positive real part, not just positive integers, but using the gamma function, which is the extension of the factorial function to the complex numbers).

Question 3

This is a difference equation, the discrete time equivalent of a differential equation. Substituting the suggested form yields expressions for k and B, and an expression for A in terms of V_0. For the second part, once you have used the binomial expansion on the bracket, the result follows.

Question 4

I think this is a little easy for STEP III – this question wouldn't be out of place on an A Level Further Maths exam. The result that the area enclosed by the curve is independent of the number of loops is nice.

Question 5

Quite a nice question about the matrix exponential. It's a neat trick to split the first sum into the Taylor series for $\sin x$ and $\cos x$.

Question 6

A bit easy for STEP III, possibly? Nothing here is particularly difficult, as long as you can remember the formula for the volume of a pyramid.

Question 7

Pretty tough. The first part is OK, and even sketching $b(x)$ is not too bad. But I think the next part with the integral is tricky, because you can't actually evaluate the integral, but you need to spot that after doing a substitution the integral (without T in it) is in fact a constant.

Question 8

I quite like this one. As always with 3D vectors questions, having a geometric intuition for what is going on is a big help. The final result from the first part is that the line is perpendicular to the radius at the point where the line and circle intersect, analogous to the same result in 2D. The third part is very quick just using geometry, but I imagine it's easy to get bogged down with vector algebra if you're not careful.

Question 9

A lot of work here! It's all pretty standard stuff which, if you have a good understanding of moments of inertia, shouldn't cause too many difficulties with figuring out *what* to do, but actually doing it requires a fair bit of persistence and accuracy.

Question 10

What horrible notation! Surely **u** for initial velocity and **v** for final velocity should have been better? Anyway, this is a nice 2D collisions question. The algebra is not too bad, as long as you can figure out the right thing to do at the right time.

Question 11

Yikes! A lot to do, with quite a lot of fiddly algebra. The first result is not too bad, with the form of the required result suggesting a conservation of energy approach. Even with the given substitution, though, the final integral is tough, with a binomial approximation and small angle approximation in there, along with various double angle formulae.

Question 12

A question about random graphs! This question is really asking if edges appear with probability p, then what is the probability that a spanning tree exists. It's a nice question, with the only real care needed in the cases that 3 or 4 roads are blocked.

Question 13

The Maximum Likelihood Estimator is an important idea in statistics, and has a number of nice properties. I think this is a tough question, because likelihood and the MLE are both new ideas, and it's made harder because the estimator must be an integer, which is what the final part of the question is about. I do like it, though!

Question 14

Another question about estimators! This is another nice one, called the German tank problem, and a classic of the genre! For a uniform distribution $U[0, N]$, the estimator Z_2 is in some sense the "best" we can do, as $Max\{X_1, \dots X_n\}$ is a *sufficient statistic*, which roughly means once we know that, we can't get any more information about the data