STEP II 1998 Comments

Question 1

I'm not great at number theory and my solution here feels very inelegant. Still, I think it works but I feel there must be a better way than reducing the problem to a finite number of candidates and showing that none of them work.

Question 2

I don't like this. It's messy, full of calculation, and not very interesting. Not much more to say, I think!

Question 3

Much better! It starts with a fairly standard method of differences problem (although care is needed, as there are three terms rather than the usual two that candidates will be familiar with). The next part follows a similar structure but with a product rather than sum – again care is needed that the end terms are dealt with correctly.

Question 4

A nice reduction formula question. The first part is fairly straightforward, just needing to use the compound angle formulae (or the product formula). Getting the correct value for $\cos(n\pi)$ as $(-1)^n$ could cause some difficulty.

Question 5

This begins with the triangle inequality, a fundamental property of normed vector spaces, which includes the complex numbers under addition. The induction proof is reasonably straightforward – establishing that the equation at the end has no solution is a bit more tricky, but careful application of the triangle inequality (and $|ab| = |a||b|$) will get you there.

Question 6

Fairly tricky, I think. Finding the gradients is not tricky, but I don't think the sketch is easy – sketching parametrically defined curves in general can be difficult. The next part is also not easy – first spotting that the normal to the first curve intersects the second curve at the point with the same parameter, then showing the normal and curve have the same gradient at this point.

Question 7

This is an analysis question – these show up occasionally. Analysis isn't really taught until undergraduate level, but well-structured questions like this are a nice way of testing candidate's ability to understand calculus and communicate their ideas clearly. Fair play to the examiners for coming up with functions which come together so nicely at the end!

Question 8

A little easy for STEP II, I think – I don't think this question (with a little more structure) would be out of place on an A Level Further Maths exam (or even an A Level Maths exam). Still, you need to have a solid understanding of vectors to solve it.

Question 9

A statics question where, as always, a good diagram is essential! Working out which rods are in compression, and which are in tension requires successive logical deductions (and also noting the symmetry of the problem across BE). Then working out the magnitude of the force in BE similarly requires starting at A (or C) and methodically resolving horizontally and vertically.

Question 10

I think this is pretty tough (not least having to distinguish between V and v throughout!). Deciding which mechanical principle to use for (i), (ii), and (iii) is not immediately obvious, and then you have to make the substitution $\dot{y} = v\sin\beta$ to solve the differential equation. As an aside, none of the papers I could find online had a copy of the diagram – luckily I had a hard copy of the original paper!

Question 11

Nothing requiring significant ingenuity here in terms of methods, but some care is needed with the algebra, especially for the second part when considering the horizontal distance between the fielder and the ball.

Question 12

A classic diagnostic test probability question, with a little more work to do for the second part. You can of course do this with a tree diagram, but you can work out the probabilities that you need without it, which speeds things up!

Question 13

A question on the memoryless property of the exponential distribution – another classic! It feels like a very basic version of queueing theory – quite an important topic in operational research. Things get much harder if your serving times are not exponential!

Question 14

A bit fiddly, particularly with reindexing the sum. I don't think it's particularly hard to figure out what to do, but actually doing it might cause some difficulties. I think the second part with the Poission distribution is easier than the first with the binomial distribution.