STEP II 1997 Comments

Question 1

This is nice. It's well structured, with the scaffolding being steadily removed and the difficulty increasing, and is also interesting. The first part of this question took me the longest – once you've got the idea of how it works, the other two parts are fairly straightforward.

Question 2

I think most of the difficulty here comes from thinking about x and y as both variables and constants at different points during the question. If one can sketch the first graph (graph sketching is an important skill for STEP!) then the rest follows fairly naturally, with perhaps the second graph another stumbling block.

Question 3

This is similar to an A Level question from the Edexcel FM Specimen paper (2017 specification). Anyway, this question is fairly short, but I do think it's not the easiest if you haven't seen something like this before.

Question 4

A neat result – and once part (ii) is completed the rest follows in the same way. A little care is needed to show that the further polynomials satisfy the same conditions on the original one – positive, distinct roots.

Question 5

The difficulty here is in setting up the integral – turning the question into mathematics. Once that is done, the integral itself is fairly straightforward to get out the right result.

Question 6

A fair bit of algebra to work through here but no real leaps of intuition to make. The only thing which slowed me down for a bit was showing that y was an even function of θ for all n, rather than being alternately odd and even as a function of x.

Question 7

Short and sweet! There are lots of proofs for the irrationality of e and this is a nice one. I think the first two parts here a fairly straightforward, with the third part becoming clear when you split the expansion of $n!\,e$ into two parts at $\frac{1}{n!}$. The final part does require some more careful thought (although hopefully the idea of using proof by contradiction is a natural one – I think most proofs of irrationality start this way).

Question 8

Yuck! A long slog through algebra, both when finding the matrix in (i) and then simplifying the diagonal elements in (iii). I'm also not sure that thinking about a change of coordinate basis would be something known to candidates back in 1997 – if not, then this question is really hard!

Question 9

A short mechanics question and reasonably straightforward – the difficulty is using the sine rule to establish $\frac{T_B}{L_B} = \frac{T_C}{L_C}$. After that it's just a question of substituting it into the expression for the ratio of the unstretched lengths.

Question 10

Another short question – the trick here is writing down the two equations of motion and using this to find a quantity which is a constant of the motion. Once that is done, it is just a case of using this in the two scenarios specified in the question.

Question 11

Yet another short one! Like many mechanics questions, the hard bit is in setting up the equations. Once you have these, actually solving them is pretty easy.

Question 12

This is a spin on the classic Coupon Collector's Problem. I like this question, up until the end where you need to find a numerical approximation for the expectation – it's messy and I think a waste of time.

Question 13

Moment generating functions are a surprisingly powerful way of thinking about probability distributions – for example showing that a sum of independent normal random variables is still normal. A fair bit of accurate algebra to work through here, but no great leaps of intuition to make, I think.

Question 14

An interesting question this one, and perhaps a tad too easy for STEP III. As long as you are confident with the formulae for expectations and variances of sums and products, then the results all fall out of the algebra. Showing that the perimeter and area are not independent requires thinking the way to do this is that $E(PA) \neq E(P)E(A)$.