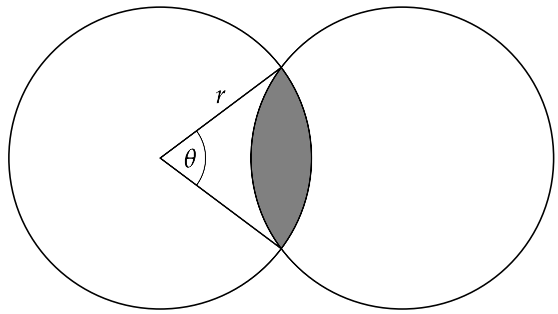
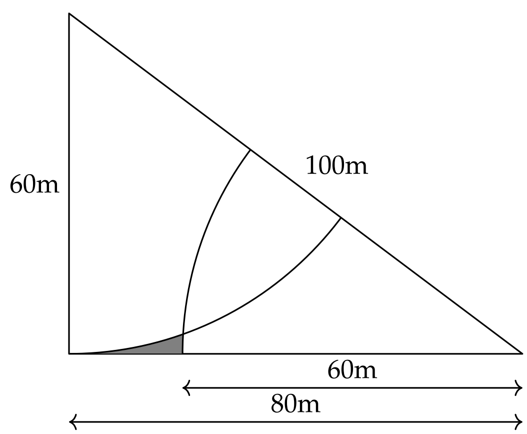
# Arcs and Sectors

**1.** Shown are two circles, both with radius $r$. The angle subtended from the centre of each circle to the points of intersection of the circles is $\theta$. Find the perimeter and area of region of intersection of the two circles, shown shaded in the diagram.



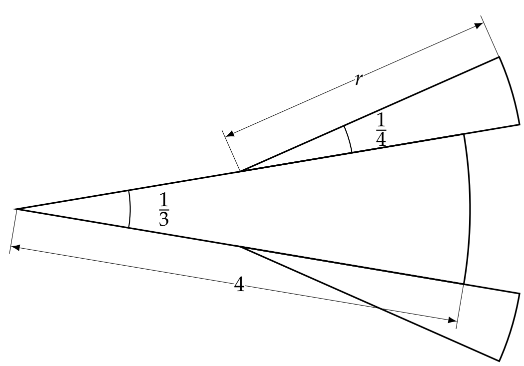

**2.** A sector of a circle has fixed area $A$, but the radius and angle may vary. Show that the minimum possible perimeter of the sector is $4\sqrt{A}$, and find the angle of the sector when this occurs.

**3.** A farmer has a field in the shape of a right angled triangle with sides 60m, 80m, and 100m. A goat is tethered to each of the two angles which are not the right angle by a rope of length 60m. Find, to three significant figures, the area of the field that cannot be reached by either goat.




**4.** The logo of a company is made up of three sectors, as shown in the diagram. The larger sector has radius 4 units and angle $\frac{1}{3}$ radians. The two smaller sectors both have radius $r$ and angle $\frac{1}{4}$ radians.

The area of the logo is $\frac{203}{48}$ units$^2$. Find the perimeter of the logo.

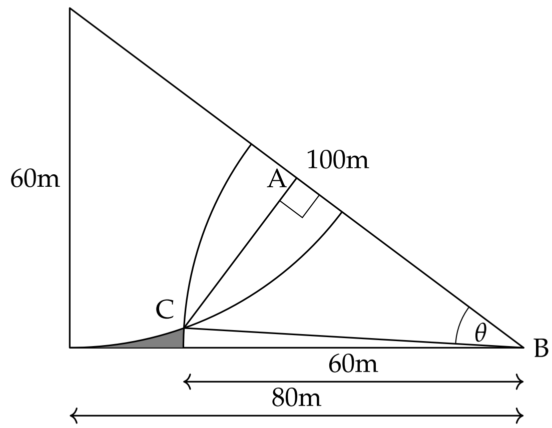



**5.** Three circles are drawn such that the centre of each circle lies on the circumference of the other two. What fraction of the total area inside this shape lies inside all three of the circles?

**1.** The perimeter is $P = 2r\theta$. The area is $A = 2 \times \frac{1}{2}r^2(\theta - \sin\theta) = r^2(\theta - \sin\theta)$.

**2.** We have $\frac{1}{2}r^2\theta = A$. So, $\theta = \frac{2A}{r^2}$. Thus the perimeter of the sector is $P = 2r + r\theta = 2r + \frac{2A}{r}$. Differentiating, $\frac{\mathrm{d}P}{\mathrm{d}r} = 2 - \frac{2A}{r^2} = 0$, so $r = \sqrt{A}$, giving $P = 2\sqrt{A} + \frac{2A}{\sqrt{A}} = 4\sqrt{A}$. Further, $\frac{\mathrm{d}^2P}{\mathrm{d}r^2} = 4Ar^{-3} > 0$, so this value is a minimum. $\theta = \frac{2A}{r^2} = \frac{2A}{A} = 2$ radians.

**3.**




As shown in the diagram, consider right-angled triangle ABC. Angle $\theta = \cos^{-1}\frac{|AB|}{|CB|} = \cos^{-1}\frac{50}{60} \approx 0.5859$. Hence the overlap of the two circles has area $60^2 \times \frac{1}{2}(2 \times 0.5859 - \sin(2 \times 0.5859)) \approx 450.15556$.

The total area enclosed by the circles is thus $\frac{1}{2} \times 60^2 \times (\tan^{-1}\frac{60}{80} + \tan^{-1}\frac{80}{60}) - 450.15556 \approx 2377.2778$.

Hence the area that cannot be reached by either goat is $\frac{1}{2} \times 60 \times 80 - 2377.2778 \approx 22.7\text{m}^2$.

**4.** The area of the logo is $\frac{1}{2} \times 4^2 \times \frac{1}{3} + \frac{1}{2} \times r^2 \times \frac{1}{2} = \frac{8}{3} + \frac{r^2}{4} = \frac{203}{48}$. Hence we obtain $r = \frac{5}{2}$.

Then the total perimeter of the logo is $2 \times 2 + 2 \times \frac{5}{2} + 2 \times \frac{1}{2} + 4 \times \frac{1}{3} + \frac{5}{2} \times \frac{1}{2} = \frac{151}{12}$ units.

**5.** The fraction inside all three circles is $\frac{\pi - \sqrt{3}}{3\pi + 2\sqrt{3}}$.