STEP II 1997 Comments

Question 1

I really like this question. On the surface it looks like a pretty standard combinatorics questions but I like the slight difference finding the sum. The fact the question asks for the answer as a product of primes is a bit of a hint to the method to use. It's nice then to go back and think about whey they chose the numbers 0, 2, 5, and 7 – in fact 0, 3, 4, and 7 would give the same answers (or 0, 1, 6, and 7 with some changes to the endpoints). I suspect that this is question that you can get very quickly or end up staring at with no progress for a while.

Question 2

A nice induction question – the main difficulty comes from successfully spotting the expression. Working out a few more terms of the sequence by hand should make it become clear. After that, it's a fairly standard induction proof.

Question 3

Pretty doable, with the structure of the question. Once you have the integrand written in partial fractions it's then just a case of splitting it into things which integrate to a logarithm and things which integrate to an arctan.

Question 4

The first part of this question is just solving a system of three linear simultaneous equations. The second part requires some ingenuity – I originally used a pretty ugly looking sum (which does work in the general case where the remainders are required to be arbitrary integers), but this solution is much nicer.

Question 5

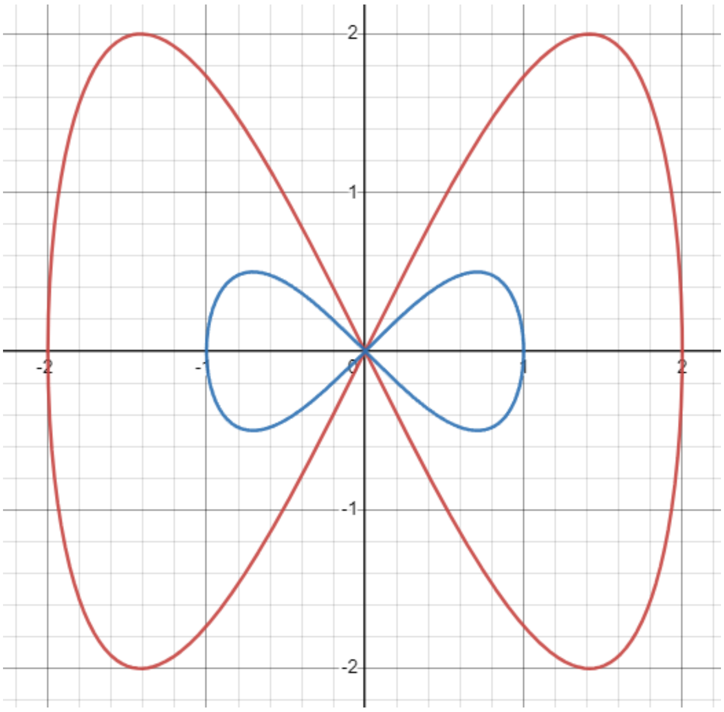
I think the hardest part here is getting your head around the relationships between the six different variables defined in this question! Once you have that the rest isn't too bad, with some care needed with the allowed values of v to make sure there is a one-to-one correspondence between the sets.

Question 6

A decent, but not particularly interesting, question. Some of the algebraic manipulation is a bit fiddly but nothing too terrible. I'm not convinced that the second part is needed – it really is very similar to the first and the question seems long enough without it.

Question 7

A nice one – using a change of variable (or just differentiating with respect to x^2) makes the first part a fair bit easier. Sketching requires some thought – but remembering that it must have reflectional symmetry along both axes (and also is smooth) helps, and once you use that and the locations of the stationary point and the zeroes the shape comes out. The actual graph is shown.



Question 8

Another one that I really like! Choosing the right function is clearly key – the $\frac{x^p}{p}$ suggest something of the form t^{p-1} – which it turns out gives the desired result. It's neat how the inequality still holds for $0 \leq x \leq 1$. Using the just established result to prove the second part is nice, and once you put the right function in the integral the result just comes out.

Question 9

As always with these sorts of questions, a good diagram is crucial! The key word in the question is "slowly" – meaning we can treat it as a statics problem. After that, the standard routines of resolving forces and taking moments along or about carefully chosen points or directions gets us the answer.

Question 10

This seems quite easy for STEP II! The tricky bit is figuring out what is going on with the cylinder, and that you can just ignore the velocity parallel to the axis. Once you've done that it just turns into an A Level question on collisions.

Question 11

I'm always impressed with the range of different projectiles questions that STEP examiners can come up with! Anyway, you follow the standard methods here and fight through some algebra to eventually get the answers out.

Question 12

A neat probability question. Well structured and a nice result at the end. The final part of the question uses the geometric distribution. STEP II 1994 Q13 also used this distribution but didn't give the result about the sum of the series needed to find the mean – I wonder if the examiners realised it is really quite hard to evaluate the sum if you haven't seen it before!

Question 13

A classic problem in probability – this is Buffon's Needle. It's often used as an example of a Monte Carlo method to approximate π (a Monte Carlo method is one which uses randomness). It turns out that is a hilariously poor way to approximate π and converges incredibly slowly. Anyway, I'm not a huge fan of this question, not because it isn't interesting but it seems unfair to include such a well known problem on a STEP paper which some candidates would have undoubtedly seen before.

Question 14

I really can't figure out the difference between the two probabilities asked for in the second part of the problem. Exactly two vehicles entering and catching up seems like the same thing as exactly two vehicles queueing behind when I leave the tunnel. I might have missed the point of this question as a result because this seems very short and not particularly interesting.