STEP I 1996 Comments

Question 1

A decent first question. Possibly a little bit on the tricky side for question 1 of STEP I, but not unreasonably so. Once you establish that $h = 2r$, it should be clear what the largest sphere to fit inside the tin is, and indeed what is the smallest sphere to contain the tin. I'm not entirely sure why they included the context of the cylinder being a biscuit tin – it doesn't add anything!

Question 2

Nice and short. I originally started by doing ii by rewriting the integral like in the first part (as is required in iii), before realising to get the binomial coefficient I would have to do the binomial expansion. It's quite a neat result and a nice way of getting there.

Question 3

I think this took me the least time out of all the STEP questions I have done so far – but that is with the benefit of lots of experience of constructing proofs at university. It's nice that each part of the question requires a slightly different method.

Question 4

Ouch! This feels much too long for a STEP I question. None of it is particularly difficult, but each has lots of algebra to plough through. I think a better question would have just been one of these integrals, possibly followed by another integral that reduces to the one already done by using a substitution.

Question 5

This was a nice question. At first glance the fact there is a connection between each part is not obvious. No major sticking points, but a fair bit of algebra to work through. Spotting that 1 is a root of the quadratic in (ii) (or -1 if you have a quadratic in p^2 rather than q^2) saves a bit of time but you can do it by the quadratic formula instead.

Question 6

A nice short one – this is a significantly easier version of a STEP III question the previous year – that is, Question 4 from STEP III 1995. No major sticking points, but noticing that the $k = 1$ case needs to be dealt with differently at the end is important, and would lead to an incorrect answer of 0 if it is ignored.

Question 7

Short and straightforward – I don't think this is much harder than an A Level question. The only slightly non-standard thing is writing e^{-kt} as b^t with $b = e^{-k}$. This might take some thinking about if you haven't seen it before but shouldn't present too much of a challenge (and is in fact how we can define powers of irrational numbers).

Question 8

Another short one! A bit more tricky than the previous question but still very doable. The tricky bit is getting the indexing for the a_i correct in the second part – it's easy to get a $+1$ or a -1 in the wrong place.

Question 9

It's nice that you can do this just by considering energy. Originally solved the last part by setting up and solving a differential equation (which does work) but this method is much nicer!

Question 10

Once you write down the three equations from total mass, conservation of energy, and conservation of momentum, it's just a case of solving these simultaneously to end up for an expression for $v_1 + v_2$ in terms of M, E, and either m_1 or m_2. You can then differentiate to find the minimum or just note the symmetry of the graph.

Question 11

A fairly standard projectiles question. Some accurate algebra is needed to go from the four original equations to the result stated in the question.

Question 12

This is a nice question – the first part is straightforward, and after that it requires some rearranging of the binomial coefficients. I think the trickiest bit is figuring out how to get rid of the k in front of the coefficient when finding the expectation – you want any term out the front to be a constant so that you can take it out of the sum.

Question 13

Another example of how e shows up everywhere in maths! The first part is straightforward but working out $E(N^2)$ is a bit more tricky. Splitting it up into two series like this is fairly short but there may be other methods too.

Question 14

Shortest STEP question ever? If you can get your head around what is going on and get α and β correct, the rest is very short and easy!