1) a) $\underline{I} = m\underline{v} - m\underline{u}$

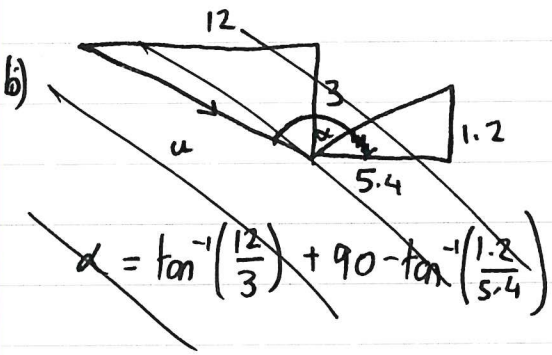
$$\begin{pmatrix}-11\\7\end{pmatrix} = m\begin{pmatrix}5.4\\1.2\end{pmatrix} - m\begin{pmatrix}12\\-3\end{pmatrix} \quad \text{M1}$$

$$\Rightarrow \begin{pmatrix}-11\\7\end{pmatrix} = m\begin{pmatrix}-6.6\\4.2\end{pmatrix} \quad \text{M1}$$

$$\frac{-11}{-6.6} = 5/3 \text{ and } 7/4.2 = 5/3. \quad \text{A1}$$

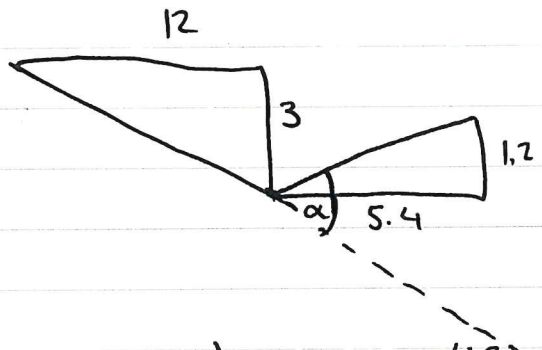
So $m = 5/3$

b)



$$\cancel{\alpha = \tan^{-1}\left(\frac{12}{3}\right) + 90 - \tan^{-1}\left(\frac{1.2}{5.4}\right)}$$

=



$$\alpha = 180 - \tan^{-1}\left(\frac{12}{3}\right) - \left(90 - \tan^{-1}\left(\frac{1.2}{5.4}\right)\right) \quad \text{M2}$$

$$= 26.6^\circ \quad \text{A1}$$

2) a) $P = Fv$

$4800 = (80+v)\cdot v$ M1

$\Rightarrow v^2 + 80v - 4800 = 0$ M1

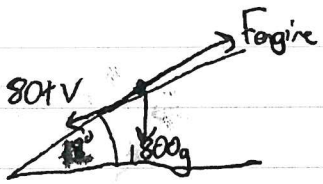
$\Rightarrow v = 40$ or $\cancel{-120}$

so $v = 40\,ms^{-1}$ A1

b) $P = Fv$

so $4800 = 40 \times F_{engine}$ M1

$\Rightarrow F_{engine} = 120N.$ A1

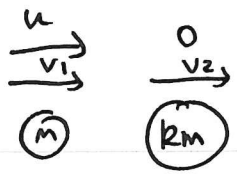


So $F_{resultant} = (80+40) + (1800g\sin 12) - 120$ M1

$= 3667.56\ldots N$ A1

$F = ma$ so $a = \frac{3667.56\ldots}{1800} = 2.04\,ms^{-2}$ A1

3) a)



Com: $mu = mv_1 + kmv_2$

$\Rightarrow u = v_1 + kv_2$ M1 A1

e: $v_2 - v_1 = eu$ M1

so $u = v_1 + kv_2$ ①

$eu = -v_1 + v_2$ ②

$keu = -kv_1 + kv_2$ ② × k

① − ② × k

$(1+e)u = (k+1)v_2$ ① + ②

$u(1-ek) = (1+k)v_1$ M1

$\Rightarrow v_1 = \frac{1-ek}{1+k}u$ A1

$\Rightarrow v_2 = \frac{1+e}{1+k}u$ A1

b) After Q collides with the wall, it has velocity $-\frac{1+e}{1+k}u$.

So there is a further collision if

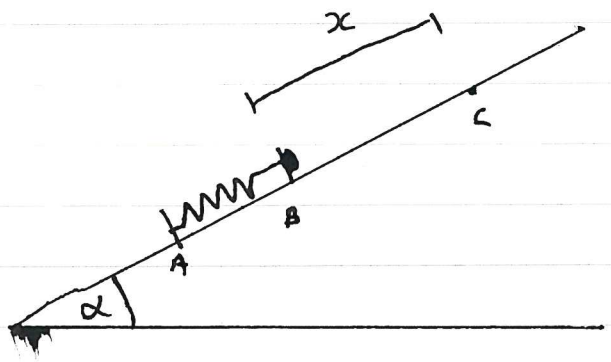
$-\frac{1+e}{1+k}u < \frac{1-ek}{1+k}u$ M1 A1

$\Rightarrow (1+e) > (ek-1)$

$\Rightarrow 2+e-ek > 0.$ A1

c) In this case, after the first collision, P is stationary and Q moves with speed u.
After the second collision between the particles, Q is stationary and P moves with speed u
A2

4a)



Energy at start is $\frac{1}{2}mv^2$

$= \frac{1}{2}m \times \frac{49}{4}g$

$= \frac{49}{8}mg$ B1

Energy at end is GPE + EPE + work done against friction

$= mgx\sin\alpha + \frac{\frac{156}{25}mgx^2}{2} + \mu mg\cos\alpha \cdot x$ M1

B1 B1 B1

$= mg\left(\frac{3}{5}x + \frac{156}{50}x^2 + \frac{2}{5}x\right)$

So $\frac{49}{8} = \frac{3}{5}x + \frac{156}{50}x^2 + \frac{2}{5}x$

$\cancel{\Rightarrow 36x^2 + 40x - 245 = 0}$

$\cancel{\Rightarrow 312x^2 + 100x -}$

$\Rightarrow 624x^2 + 200x - 1225 = 0$ M1

$\Rightarrow x = 5/4$ or $\cancel{-\frac{245}{130}}$

so $x = 5/4$ metres A1

~~b) Stationary, so friction = 0~~

~~$F = \frac{x}{l} + \mu mg\cos\alpha$~~ B1 B1

~~$= \frac{156}{25}mg \cdot \frac{5}{4} + \frac{2}{5}mg$~~

~~$= \frac{41}{5}mg.$~~ A1

~~$F = ma$, so $\frac{41}{5}mg = ma$~~ M1

~~$\Rightarrow a = \frac{41}{5}g$~~ A1

b) $F = mg\sin\alpha + \frac{\lambda x}{l} - \mu mg\cos\alpha$ M1 M1

$= \frac{3}{5}mg + \frac{156}{25}mg \cdot \frac{5}{4} - \frac{2}{5}mg$

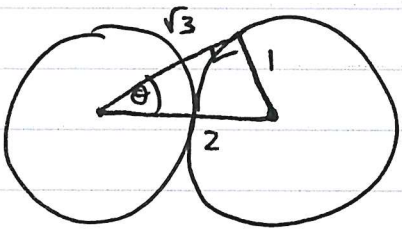
$= 8mg$ A1

$F = ma$, so $8mg = ma$

$\Rightarrow a = 8g$ M1

5)

a)



$\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30°$. M1

So the initial velocity of A is $\begin{pmatrix} u \\ \frac{1}{\sqrt{3}}u \end{pmatrix}$ A1

The final velocity of A is $\begin{pmatrix} v \\ \frac{1}{\sqrt{3}}u \end{pmatrix}$ and the final velocity of B is $\begin{pmatrix} w \\ 0 \end{pmatrix}$ M1

Considering the horizontal direction

CoM: $mu = mv + kmw$ M1

$\Rightarrow u = v + kw$

e: $eu = w - v$ M1

Because A is moving NE, we have $v = \frac{1}{\sqrt{3}}u$ M1

$\Rightarrow u = \frac{1}{\sqrt{3}}u + kw$

and $\frac{1}{\sqrt{3}}u = w - \frac{1}{\sqrt{3}}u$

$\Rightarrow w = \frac{2}{\sqrt{3}}u$ M1

so $u = \frac{1}{\sqrt{3}}u + \frac{2}{\sqrt{3}}wk$

$\Rightarrow u\sqrt{3} = u + 2wk$

$\Rightarrow \sqrt{3} = 1 + 2k$

$\Rightarrow k = \frac{\sqrt{3}-1}{2}$ A1

b) <u>Initial KE</u>

$\begin{pmatrix} u \\ \frac{1}{\sqrt{3}}u \end{pmatrix}$

$\Rightarrow \quad v^2 = u^2 + \frac{1}{3}u^2$ M1

$= \frac{4}{3}u^2$

so $KE = \frac{1}{2}mv^2$

$= \frac{2}{3}mu^2$ B1

<u>Final KE</u>

velocity of A is $\begin{pmatrix} \frac{1}{\sqrt{3}}u \\ \frac{1}{\sqrt{3}}u \end{pmatrix}$

$v^2 = \frac{1}{3}u^2 + \frac{1}{3}u^2$

$= \frac{2}{3}u^2$

so $KE = \frac{1}{3}mu^2$ B1

velocity of B is $\begin{pmatrix} \frac{2}{\sqrt{3}}u \\ 0 \end{pmatrix}$

$v^2 = \frac{4}{3}u^2$ M1

$KE = \frac{2}{3}u^2 m \cdot \left(\frac{\sqrt{3}-1}{2}\right)$

$= \frac{\sqrt{3}-1}{3}mu^2$ B1

So fraction lost $= \dfrac{\frac{2}{3}mu^2 - \frac{1}{3}mu^2 - \left(\frac{\sqrt{3}-1}{3}\right)mu^2}{\frac{2}{3}mu^2}$ M1

$= \dfrac{2-1-\sqrt{3}+1}{2}$

$= \dfrac{2-\sqrt{3}}{2}$ A1

c) e.g. sound, heat

6) Afterwards, the particle must be travelling with velocity $u\begin{pmatrix}-1\\1\end{pmatrix}$ or $u\begin{pmatrix}-1\\-1\end{pmatrix}$ for $u>0$

M1

Now $I = mv - mu$ M1

$= 3u\begin{pmatrix}-1\\1\end{pmatrix} - 3\begin{pmatrix}3\\0\end{pmatrix}$ or $3u\begin{pmatrix}-1\\-1\end{pmatrix} - 3\begin{pmatrix}3\\0\end{pmatrix}$

$= 3\begin{pmatrix}-u-3\\u\end{pmatrix}$ or $3\begin{pmatrix}-u-3\\-u\end{pmatrix}$

So $|I|^2 = 3^2\left[(u+3)^2+u^2\right]$ M1

$= 9\left[2u^2+6u+9\right]$

So $45^2 = 9\left[2u^2+6u+9\right]$ M1

$\Rightarrow 225 = 2u^2+6u+9$

$\Rightarrow 2u^2+6u-216=0$

$\Rightarrow u = 9$ or $\cancel{-12}$

So $u=9$. A1

So $I = 3\begin{pmatrix}-9-3\\9\end{pmatrix}$ or $3\begin{pmatrix}-9-3\\-9\end{pmatrix}$

$= \begin{pmatrix}-36\\27\end{pmatrix}$ $= \begin{pmatrix}-36\\-27\end{pmatrix}$

So $\lambda=-36, \mu=27$ or $\lambda=-36, \mu=-27$

A1 A1

7) a) Initial velocity of particle is $\begin{pmatrix} u \\ -u\tan\theta \end{pmatrix}$ M1

After the first collision, velocity is $\begin{pmatrix} u \\ eu\tan\theta \end{pmatrix}$ M1

After the second collision, velocity is $\begin{pmatrix} -eu \\ eu\tan\theta \end{pmatrix}$ M1

After the third collision, velocity is $\begin{pmatrix} -eu \\ -e^2u\tan\theta \end{pmatrix}$ M1

For this to be perpendicular,

$$\begin{pmatrix} u \\ -u\tan\theta \end{pmatrix} \cdot \begin{pmatrix} -eu \\ -e^2u\tan\theta \end{pmatrix} = 0 \quad \text{M1}$$

$$\Rightarrow -eu^2 + e^2u^2\tan^2\theta = 0$$
$$\Rightarrow -1 + e\tan^2\theta = 0$$
$$\Rightarrow e\tan^2\theta = 1 \quad \text{A1}$$

b) $\tan^2\theta = \frac{1}{e}$, and $e \leq 1$ M1, so $\tan^2\theta \geq 1$

$$\Rightarrow \tan\theta \geq 1$$
$$\Rightarrow \theta \geq 45^\circ \quad \text{A1}$$

c) After the third collision the $\underline{i}$ speed is eu and the $\underline{j}$ speed is $e^2u\tan\theta = e^2u\sqrt{3}$
After every subsequent collision, the $\underline{j}$ speed reduces by a factor of e. So after n collisions, the $\underline{j}$ speed is $e^{n-2}u\sqrt{3}$. A1

So the ratio of speeds is $\dfrac{e^{n-2}u\sqrt{3}}{eu} = e^{n-3}\sqrt{3}$. A1 But $e = \dfrac{1}{\tan^2\theta} = \dfrac{1}{3}$. B1

So ratio is $\left(\frac{1}{3}\right)^{n-2}\sqrt{3}$ A1

d) The angle φ between $\underline{i}$ and particle motion satisfies $\tan\varphi = \left(\frac{1}{3}\right)^{n-2}\sqrt{3}$ M1

so $\left(\frac{1}{3}\right)^{n-2}\sqrt{3} \leq \tan(1^\circ)$ M1

$$\Rightarrow \left(\frac{1}{3}\right)^{n-2} \leq \frac{\tan(1^\circ)}{\sqrt{3}} \Rightarrow (n-2) \geq \cancel{\frac{\tan(1^\circ)}{\sqrt{3}\log\left(\frac{1}{3}\right)}} = \frac{\log\left(\frac{\tan(1^\circ)}{(\sqrt{3})}\right)}{\log\left(\frac{1}{3}\right)} = 4.18 \quad \text{M1}$$

$$\Rightarrow \underline{n = 7} \quad \text{A1}$$