Further Mechanics Practice Paper 1

1. A particle of mass m kg is travelling with velocity $12\mathbf{i} - 3\mathbf{j}$ ms^{-1}. It receives an impulse of $-11\mathbf{i} + 7\mathbf{j}$ Ns. After this impulse, the particle travels with velocity $5.4\mathbf{i} + 1.2\mathbf{j}$ ms^{-1}.

a) Show that the mass of the particle is $\frac{5}{3}$ kg.

(3)

b Find the angle of deflection of the particle, giving your answer in degrees to one decimal place.

(3)

2. A car of mass 1800 kg is travelling along a horizontal road. The resistance to motion of the car is modelled as a variable force of magnitude $(80 + v)$N. The engine of the car is working at a rate of constant rate of 4.8 kW.

a) Find the maximum possible speed of the car.

(3)

b) While the car is travelling at its maximum possible speed, it starts driving up a hill, inclined at 12° to the horizontal. Find the deceleration of the car the instant that it begins driving up the hill.

(5)

3. Two particles P and Q, of masses m kg and km kg respectively, move along a straight line on a horizontal table. The coefficient of restitution between the two particles is e. Initially, Q is at rest and P moves with speed u ms^{-1} in the direction of Q. The particles collide directly. After the collision, the direction of motion of P is reversed.

a) Show that, after the collision, in the direction PQ the velocity of

i) P is $\frac{1-ek}{1+k}u$

ii) Q is $\frac{1+e}{1+k}u$

(6)

b) After the collision, Q collides directly with a vertical wall. The collision is perfectly elastic. Show that there is a further collision between P and Q only if

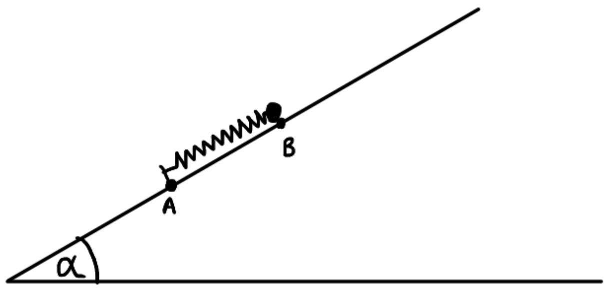
$$2 + e - ek > 0$$

(3)

c) Explain briefly the motion in the case $e = k = 1$.

(2)

4.



A light elastic spring has natural length 1 metre and modulus of elasticity $\frac{156}{25}mg$. The spring has one end fixed at A to a rough horizontal plane. The other end of the spring is attached to a particle of mass m kg, which is initially at point B, where AB = 1 metre. The plane is inclined at an angle α to the horizontal, where $\tan\alpha = \frac{3}{4}$. The coefficient of friction between the particle and the plane is $\frac{1}{2}$. The particle is projected up the plane with speed $\frac{7}{2}\sqrt{g}$ ms^{-1}. The particle first comes to rest at a point C.

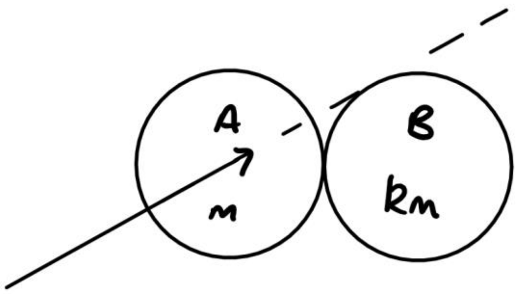
a) Find the distance BC.

(7)

b) Find the acceleration of the particle when it is at C, giving your answer in terms of g.

(4)

5. In this question, the vector **i** is a unit director pointing east, and the vector **j** is a unit vector pointing north.



A sphere B of unit radius and mass km is at rest on a horizontal plane when it is struck by a sphere A, also of unit radius but of mass m, such that before the impact, the direction of motion of the centre of A is tangential to the sphere B, as shown in the diagram. The coefficient of restitution between the spheres is $\frac{1}{\sqrt{3}}$.

After the collision, the sphere A is moving in a north-east direction.

i) Show that $k = \frac{\sqrt{3}-1}{2}$

(8)

b) Show that the fraction of kinetic energy lost in the collision is $\frac{2-\sqrt{3}}{2}$.

(7)

c) State one way this energy could have been released.

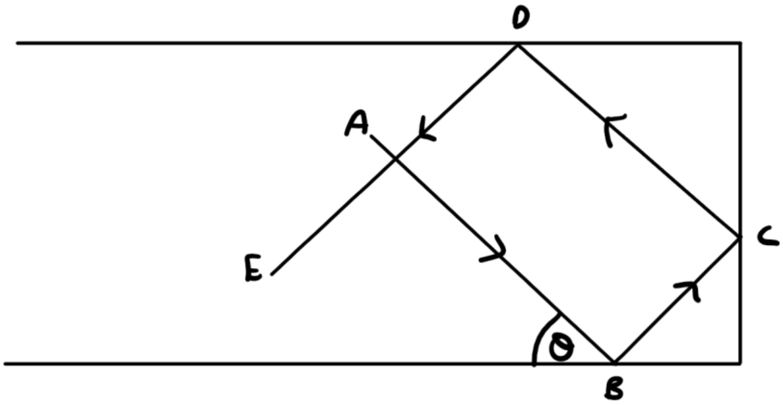
(1)

6. A particle of mass 3 kg is moving with velocity $3\mathbf{i}$ ms^{-1}. The particle receives an impulse $\mathbf{I}$ of magnitude 45 Ns. After the impulse, the direction of motion of the particle is deflected through an angle of 135°.

Given that $\mathbf{I} = \lambda\mathbf{i} + \mu\mathbf{j}$, find the possible pairs of values of λ and μ.

(7)

7.



A particle moves along the horizontal floor of a long rectangular room, as shown in the diagram. It collides repeatedly with the vertical walls, which are at right angles to each other, in the order ABCDE. The angle between the particle's initial velocity and the first wall it collides with is θ. The coefficient of restitution for all collisions is e.

After the third collision (on the line DE), the velocity of the particle is perpendicular to its initial velocity (on the line AB).

a) Show that $e \tan^2 \theta = 1$

(6)

b) Hence conclude that for this motion to be possible, we must have $\theta > 45°$

(2)

c) The particle continues to collide repeatedly with the walls. The room is long enough such that all collisions are with the walls parallel to $\mathbf{i}$. Given now that $\theta = 60°$,

i) Show that, after n collisions, the ratio of the speed in the $\mathbf{j}$ direction to the $\mathbf{i}$ direction is $\left(\frac{1}{3}\right)^{n-2}\sqrt{3}$

(4)

ii) Hence find after how many collisions the angle between the direction of travel of the particle and $\mathbf{i}$ is less than one degree.

(4)