STEP III 1995 Comments

Question 1

A nice first question. The equations are set up so it is very easy to eliminate two variables at once to find expressions for x, y, and z. The important thing is to keep track of what is being divided at each step, making a not to later check what happens when these quantities are zero.

Question 2

Choosing a sensible substitution is necessary to do the first part ($x = a\cos^2 u$ would also work, introducing a minus sign but also switching the limits). Once that is done, it's just a straightforward application of the double angle formulae. The second part is a fairly standard reduction formula using integration by parts, and after writing out the first few I_n, the general form should become clear.

Question 3

Quite a nice result. I think the trickiest bit here is spotting how to simplify the expression for $\left|\frac{x_{n+1}}{x_n}\right|$ using the periodicity of sine. A common pitfall (which would still, incorrectly, lead to the correct answer) is to use the form $\sin\sqrt{k^2-1}\,t$. $0 < k < 1$, so this is imaginary.

Question 4

I found this very fiddly. I think this is probably the most efficient method; you can also write C_n as the real part of a geometric series using De Moivre's theorem. I have very rarely seen the sum and product formulae for sine and cosine outside of STEP questions and this did feel fairly contrived.

Question 5

I liked this one. It is all fairly standard and just requires patience working through the algebra. The hardest bit is coming up with the expression for general term of the Maclaurin series. If you haven't seen Π to write a product before then you would struggle.

Question 6

This is a nice question. The first part took me longer than it should have done, but after that the rest followed without too much difficulty. Figuring out the locus of w is the hardest bit I think – I worked out the coordinates for some judiciously chosen values of t and then hypothesised it would be a circle (using the intuition from the first part), and then showed that it worked.

Question 7

Groups questions always feel a bit strange. I breezed through this with the benefit of undergraduate group theory, but I don't know how much group theory a STEP candidate in 1995 would have studied. As such, it's hard to say how difficult this question would have been.

Question 8

Again, a question that I found fairly straightforward, but I think would be very tricky without the benefit of some undergraduate vectors. No knowledge beyond A Level needed, but taking a given equation and dotting it with something else is something I didn't see until university. Establishing and dealing with the $\pm$ takes some care. I also thought it was OK to justify $x = y$ by pretty much saying "it's obvious". I don't think they want a lengthy proof here but I might be wrong.

Question 9

Long! The last part in particular, maximising v^2, felt entirely unnecessary. Apart from that, a decent question, although the start felt like a lot of geometry rather than any mechanics. It had been a while since I'd done any moments of inertia so I did have to spend a bit of time reading about those before I could do the question.

Question 10

A nice question. Nothing particularly non-standard, although realising that you can maximise x rather than $d = \sqrt{x^2 + y^2}$ makes things a lot simpler! Some use of the trig product formulae but after Question 4 I felt like an expert!

Question 11

Fairly long, but doable. Making sense of the question at the start is the hardest bit – once you have an expression for the velocity of the plane relative to the air, the rest follows fairly straightforwardly. I used the fact that the fraction of time on the outwards journey is $\frac{u_{back}}{u_{out}+u_{back}}$ which perhaps isn't intuitively obvious, but you can get there with a few more lines of algebra with some simultaneous equations.

Question 12

Short, but tricky. The first part is not too bad, the main difficulty being splitting the original probability into the two cases of $V < 0$ and $V > 0$. After that, I think it gets a bit more fiddly, noting that fact that the fact the density is radially symmetrical means that the probability of being in a sector of angle α is $\frac{\alpha}{2\pi}$, and justifying that the method still works for negative k.

Question 13

I found this a bit fiddly, with all the ps, qs, αs and βs flying around. Some of the parts took a bit of time to parse and make sense of, as well. The actual calculations were fairly straightforward. It is important to consider the possibility of two errors cancelling each other out for the final part.

Question 14

This was a fairly nice question. A fair bit of algebra to work through but the integration is straightforward when you do it by parts. I'm unsure if I actually needed to prove the result in the final part of the question. The question says "find".