## STEP III 1994 Comments

## Question 1

A pretty nice start. The only possible stumbling block are spotting the right substitution to do the first integral, and splitting the $\operatorname{sech}^{n} t$ into $\operatorname{sech}^{2} t$ and $\operatorname{sech}^{n-2} t$ for the integration by parts. I quite like reduction formulae in general, it feels like quite a neat way to evaluate integrals.

## Question 2

Classic STEP structure - follow a given method, figure out why it worked, and apply it to a harder problem. A few stumbling blocks here - dividing by $x^{2}$, completing the square, and also spotting what substitution to use for the second equation. Overall, a pretty reasonable STEP III question.

## Question 3

I found this pretty straightforward but that is with the benefit of having studied vectors in detail at university. I think if you aren't confident working with 3D vectors, this is tricky. The concept of the circles being linked is almost certainly new to the candidate and requires being able to visualise the situation well.

## Question 4

This one felt a bit weird. After you figure out how to solve the differential equation, there is nothing else too tricky, but it is important to keep track of what all your different letters represent - variables, arbitrary constants, fixed constants that you are trying to find, or fixed constants that you don't know.

## Question 5

Started off pretty nice, with a neat proof by induction. The final part comparing the expansion to that of $g$ felt a bit unnecessary and also a bit tricky.

## Question 6

Quite a nice bit of maths, but the end of the second part felt quite tricky. I'm quite curious about whether the second result holds when the points are not on the unit circle, and also whether this extends to $n$ points needing to be the vertices of a regular $n$-gon for the result to hold.

## Question 7

Some fairly elementary group theory going on here. I don't know how much group theory was on the A Level specification back in 1994 so it's hard to say how difficult students might have found this question.

## Question 8

I found this a tricky one, with a few leaps of intuition required. There was lots of algebra to work through as will, with a requirement to be accurate, particularly when working out which sign the answer should be with all the is floating around.

## Question 9

A nice question. Some work is required to get to the differential equation, but once you notice that it is SHM the rest of the question is quite pleasant with a nice result at the end.

## Question 10

This was a vectors and geometry question, rather than a mechanics question. I found the first part pretty straightforward but got pretty bogged down with angles for the second part before spotting how to do it more easily! Overall tricky, but not too long.

## Question 11

A hard, but doable, statics question. Nothing out of the ordinary, but the difficulty is getting all of the correct forces on the diagram and then choosing the correct directions to resolve forces and where to take moments. The algebra isn't too tricky but does require care.

## Question 13

Finding $p_{\text {short }}$ requires thinking recursively, and then finding $p_{\text {stan }}$ requires noticing that if the game reaches $2-2$, it just reverts to a game of short tennis. I had a bit of trouble ploughing through the algebra at the end - there may be a much more efficient way of doing it!

## Question 14

A nice question, first demonstrating the memoryless property of the exponential distribution, and then working out some probabilities involving this distribution. The final bit involving the sum isn't something l've seen before and was nice.

## Question 15

This is hard. Far harder than I think is fair for a STEP question, even the final question on a STEP III paper. That said, it is a really nice question and one that I am surprised I haven't come across before.

