## STEP II 1994 Comments

## Question 1

I didn't really like this one. I found it quite tricky and it took me a while to figure out how to do it. I'm not sure the result is particularly interesting either. So not one of my favourite questions!

## Question 2

A nice question! Felt like fairly classic STEP - a fair bit of algebra, spotting and extending a pattern (with the repeated integration by parts), and the whole question is about a particular mathematical object that will be unfamiliar to the candidate but familiar to undergraduate mathematicians (in this case, the Legendre polynomials).

## Question 3

The difficulty here is choosing the right $x$ and $y$ for each part of the question. I made a couple of false starts on a couple of them but made it through without too much trouble. For the final part of the question, once you have the insight that $f(x)=\cos x$ fulfils the criteria, the rest follows.

## Question 4

I liked this! I hadn't seen the method to find the area enclosed in an ellipse by using a substitution to turn it into a circle (although it seems obvious in hindsight!). Some good diagrams are necessary here.

## Question 5

Pretty doable, I think. Adding the two modulus functions to get a graph with a horizontal section led me down a bit of a rabbit hole later thinking how you could approximate a function in a certain neighbourhood using a combination of various modulus functions. The final part of the question felt like it had a bit of a different flavour to the rest.

## Question 6

A pretty standard "do as you are told" question here - the induction is reasonably straightforward. The second part is not a problem as long as you can recognise how to use the small angle approximation for tan.

## Question 7

Yuck! I didn't like the amount of algebra to plough through on this question. The result in the end was vaguely interesting but not enough to justify the pain of working it all out!

## Question 8

Hmm, I'm not sure what to think about this one. It feels quite different to most other STEP questions in the context. Once you have figured out how to turn it all into algebra (and got the correct expression for the storage costs) the rest is just working through the algebra.

## Question 9

I found this one really tough. Don't be fooled by my neat working out - I ended up drawing out the diagram five times when I was working through the question for the first time - the final one was as big as half a page of A4! I initially started by resolving forces vertically and horizontally and taking
moments at $A$ and $B$ before realising it was really a geometry problem rather than a mechanics problem.

## Question 10

Short but tough! I think relating $x$, the extension of the rope to the position of the trailer relative to the truck is a bit of a leap of intuition, and then writing down the differential equation is another one. Once that is done, the rest follows without too much trouble.

## Question 11

Long but pretty doable. Nothing which requires much ingenuity, just ploughing through the algebra. Establishing the $u \leq v$ solution is maybe the only bit which needs some thought. None of the three mechanics questions on this paper were particularly nice, I think!

## Question 12

Nothing too terrible here. It's nice when you spot why the numbers in the question where chosen! Some careful use of conditional probability is needed to get both of the results. I thought the given answer "hides" the fact that each of the probabilities in the brackets is actually a fraction our of 36 quite well.

## Question 13

This feels like a standard first year undergraduate probability question (and indeed this is the coupon collector's problem, a fairly standard problem). I think if you haven't seen the trick to find the mean of the geometric distribution before, that is very hard to come up with in an exam. Establishing the inequality isn't too bad but I think using that to obtain the asymptotics for the mean is tricky if you haven't done anything like that before. In fact the expectation is asymptotically $n \log n+\gamma n+\frac{1}{2}$, where $\gamma$ is something called the Euler-Mascheroni constant.

## Question 14

A straightforward question if you are confident dealing with continuous random variables, and almost impossible if not! I liked the first part of the question but thought the second part was a bit naff, having to resort to a calculator to find the optimal solution!

