STEP I 1995 Comments

Question 1

A nice first question. Each part of the question requires a bit of clever thinking. Spotting that $(x-4)$ is a factor of the first inequality helps for the first part, then spotting the link between the first inequality and second equation, then just choosing a point to test in one of the regions.

Question 2

Both parts of this question are unrelated, which is unusual. I found the hardest part figuring out how to evaluate $S-T$, before spotting it is in the form $f'(x)/f(x)$. The second part is pretty doable with the given substitution, with everything falling into place with a couple of trig identities.

Question 3

If you have seen the method of differences before, the first two parts are just bookwork. The third part does require some more thought, noticing that the sum of the even cubes up to $2n$ is 8 times the sum of the cubes up to n. After that, the rest follows.

Question 4

Another nice question. It does make me wonder if this use of De Moivre's theorem was on the specification back in 1995 – today the first part of this question is just a standard further maths A Level question. The second part requires some more thought, but nothing too bad.

Question 5

Doable, but with some stumbling blocks. The first part requires some careful algebra. The substitution to use for the second part should be fairly clear. Correctly factorising $1-y^n$ in the last part is probably the hardest bit, unless it has been seen before.

Question 6

Classic STEP – use a given method to solve a first problem, and adapt this method to solve a similar problem. In this case, it should be fairly clear how to adapt the first method. Overall I thought a pretty straightforward question.

Question 7

I started off with a diagram here, but it turned out to not be very useful. A bit of a clear head is needed to get all the formulae for midpoints etc. correct with minus signs in the right places, but apart from that it's just a bit of working through.

Question 8

An algebra-fest here! No major stumbling blocks apart from the need to be accurate throughout. At first this seems quite strange, because it seems like we have three linearly independent solutions to a second order differential equation, which shouldn't happen. But as it turns out, the general solution is $y = A\left(\frac{1}{x} - 1\right) + B(1-x) + 1$.

Question 9

This was OK – I think there are probably other methods that work as well but this one felt most intuitive to me. It's a bit weird differentiating with respect to θ and keeping x constant is an unusual one (and actually is partial differentiation, a first year undergraduate topic).

Question 10

Quite a lot to do for a STEP I mechanics question, I think. It took me a bit of time to come up with the link between x and y. It's interesting that in the limit as the number of bounces goes to infinity, the speed at $x = 0$ tends to $\frac{\lambda}{lm}h^2$, independent of the initial speed.

Question 11

A tricky one – a good diagram is crucial. It took me a while to figure out how to include the frictional forces between the spheres (and in which direction they acted). Once that's done, it involves the standard statics methods of resolving forces and taking moments, eventually leading to the required inequality.

Question 12

Nice and short! It does require some thought, particularly for the third part.

Question 13

Another bit of a slog through the algebra. The second part in particular felt quite fiddly, especially introducing extra terms to get the factorised form as required. The first part of the question is an important result in probability, known as the thinning property of a Poisson process. The second part is not particularly interesting.

Question 14

Hmm, I'm not that keen on this one. It wouldn't get asked today, now that calculators and tables are not permitted. It just feels a bit messy.