STEP I 1994 Comments

Question 1

A nice first question. The trickiest bit is probably interpreting the wording in the question into a mental model of a 3D shape. After that, we just need to use standard results for volume and area of various shapes.

Question 2

A nice straightforward one. If you can recall the standard method (implicit differentiation) to find the derivative of a^x, the rest follows naturally using the same method.

Question 3

No huge leaps of intuition needed here, but a solid understanding of binomial coefficients and confidence in combining the terms from two different expansions is required.

Question 4

The first part of this question is a straightforward application of the compound angle formulae. The second part is also straightforward through completing the square. The third part is not too long but does require some good thinking of how to deal with the two arctan terms.

Question 5

This feels like a classic STEP I question. No significant "challenge" element but an application of coordinate geometry requiring a solid understanding of the A Level content and also strong algebra skills.

Question 6

This question requires some careful algebraic manipulation but is fairly straightforward otherwise. In the real numbers, self-inverse functions exist (like $f(x) = k/x$), and it is nice for candidates to see than in the complex numbers we can have functions which must be composed with themselves multiple times (in this case three) to get the identity function.

Question 7

I liked this question. Spotting the pattern is straightforward but turning this into an algebraic statement requires some care. Once this is stated correctly it is not too tricky to prove using the standard result given at the end of the question. The second part requires a bit more ingenuity in how to rewrite the sum in a way that the result established in the first part can be used.

Question 8

A nice integration question. Using the given substitution leads to the result, with some algebraic manipulation required on the function inside the logarithm. It also requires a recognition that multiplying by -1 allows you to switch the limits on the integral. The second part requires some thought to pick a sensible substitution, but the denominator of $1 + x^2$ suggests that $x = \tan\theta$ might be a good choice. For the final part, the key insight is the recognition that you can rewrite $\cos(x)$ as $\sin\left(\frac{\pi}{2} - x\right)$. I thought this was probably the hardest pure question on the paper.

Question 9

A bit of an algebra slog! After noticing you can use the Pythagorean trig identity, the first part of the question is reasonably straightforward. After that, it is important to realise that we now need to substitute back in for θ and eliminate t and then a similar method to previously follows.

Question 10

A fairly standard start here, resolving forces horizontally and vertically. One an expression for $\cos\theta$ is established, the tricky bit is spotting that this allows us to establish the inequality.

Question 11

This is a standard A Level mechanics questions, just with algebra instead of numbers!

Question 12

A question involving the standard rules for conditional probability. Some care is needed to turn the worded questions into algebraic probabilities correctly.

Question 13

A tricky question, I think. It requires some confidence with probability (for example, being able to assert that the expectation of each T_i is the same). Part (iv) is the hardest, and once that result is established, (v) follows fairly straightforwardly.

Question 14

A nice question exploring some of the properties of the exponential distribution. The only real challenge comes from being able to write down the cumulative distribution functions of T and U by writing them in terms of the T_i and splitting these up using independence.