Core Pure Extra Practice Paper 2

1.

$$z_1 = 1 - i$$

$$z_2 = 3\sqrt{3} + 3i$$

a) Find the values of

i) $\arg(z_1)$

ii) $\arg(z_2)$

iii) $\arg(z_1 z_2)$

(3)

b) Find the value of $|z_1 + z_2|$, giving your answer in the form $\sqrt{a + b\sqrt{c}}$, where a, b, and c are integers.

(2)

2.

$$M = \begin{pmatrix} 1 & 5 & p \\ q & 1 & 4 \\ 0 & 0 & 1 \end{pmatrix}$$

a) Find the values of p and q for which the matrix M has an inverse.

(3)

b) Given that M has an inverse, find M^{-1} in terms of p and q.

(3)

c) Write down the determinant of M^{-1}, giving your answer in terms of p and/or q.

(1)

d) The matrix M represents the linear transformation A.

Given that,

- The determinant of M is positive,
- Under A, a cube with side length 2 is mapped to a shape with volume 48 units^3,
- $(8, -4, 2)$ is an invariant point of the transformation,

Find the values of p and q.

(4)

3.

The curve C has equation

$$y = 5\cosh x \sinh x - 12\cosh 2x$$

a) Find the x coordinate of the stationary point A of C, giving your answer exactly in terms of natural logarithms.

(3)

b) Show that A is a maximum turning point.

(2)

4.

a) Express

$$\frac{1-2x}{x^4-2x^3+x^2}$$

in partial fractions.

(4)

b) Hence show that

$$\sum_{r=k+1}^{n} \frac{2r-1}{r^4-2r^3+r^2} = \frac{n^2-k^2}{n^2k^2}$$

(2)

c) Find the least value of n such that

$$\sum_{r=6}^{n} \frac{2r-1}{r^4-2r^3+r^2} > 0.039$$

(3)

5.

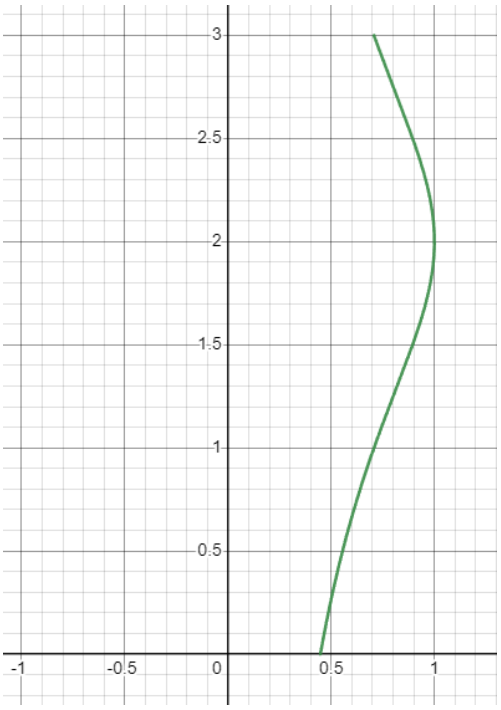
i) Sketch the polar curve $r = e^{-\theta}$ for $0 \leq \theta < 2\pi$

(1)

ii) Find the points on the curve where the tangents are perpendicular to the initial line. Give your answers correct to three significant figures.

(4)

6.



The diagram shows the curve $x^2(1 + (y - 2)^2) = 1$ for $x \geq 0$, $0 \leq y \leq 3$.

a) Find the volume of the solid generated when the curve is rotated 2π radians around the y axis, giving your answer to three significant figures.

(3)

b) Find the area enclosed by the curve, the y axis, the x axis, and the line $y = 3$, giving your answer in the form $\ln \frac{a+\sqrt{b}}{c+\sqrt{d}}$.

(3)

7.

In an argand diagram, ABCDEF is a regular hexagon centred at the origin, where $A = \sqrt{3} + i$.

The locus of points z satisfying $|z| = |z - \sqrt{3} - i|$ divides the hexagon into two regions.

Find the ratio of the area of the larger region to the area of the smaller region.

(6)

8.

Two planes are defined by

$$\Pi_1: -5x + 6y + z = -52$$

$$\Pi_2: 3x - 4y + 2z = 24$$

a) Find the acute angle between Π_1 and Π_2, giving your answer to one decimal place.

(3)

b) Find a vector equation of l, the line of intersection of the two planes.

(4)

c) The point A lies on l. Find the coordinates of A when the distance from A to the origin is minimised.

(4)

9.

The displacement of a particle from a fixed origin is modelled by the differential equation

$$\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 9x = -4e^{-3t}$$

a) Find the general solution of the displacement of the particle at time t.

(6)

b) Initially, the particle has displacement $\frac{5}{3}$ and is stationary.

Find the other time when the particle is stationary.

(5)

c) Find

$$\int_0^\infty x\, dt$$

(6)