Core Pure Extra Practice Paper 1

1.

$$f(z) = z^3 + pz^2 - 5z + q$$

Where p and q are real constants.

Given that $z = -3 + 2i$ is a root of $f(z) = 0$, find all three roots of the equation and hence the values of p and q.

(8)

2.

Consider the recurrence relation

$$u_{n+2} = 2u_{n+1} + 15u_n \ (n \geq 1)$$

with $u_1 = -7$ and $u_2 = 61$.

Prove by induction that $u_n = 4 \times (-3)^n + 5^n$ for $n \geq 1$.

(6)

3.

For each of the following integrals, determine if it is convergent. If it is convergent, find its value.

i)

$$\int_0^\infty \frac{1}{1 + x^2} \, \mathrm{d}x$$

(2)

ii)

$$\int_{-1}^1 \frac{1}{x} \, \mathrm{d}x$$

(3)

iii)

$$\int_0^8 \frac{1}{\sqrt[3]{x}} \, \mathrm{d}x$$

(2)

4.

The number of bacteria in a petri dish is modelled using the differential equation

$$\cosh t \frac{dN}{dt} = \sinh t \,(\sinh^3 t \cosh t - N)$$

where t is the time in days after the start of the study, and N is the number of **millions** of bacteria.

Initially, there are 500,000 bacteria.

i)

Find, according to the model, the number of bacteria in the petri dish 18 hours after the start of the study. Give your answer to three significant figures.

(6)

ii)

State a limitation of the model.

(1)

5.

i)

Use De Moivre's theorem to prove that

$$\cos(6\theta) = 32\cos^6\theta - 48\cos^4\theta + 18\cos^2\theta - 1$$

(5)

ii)

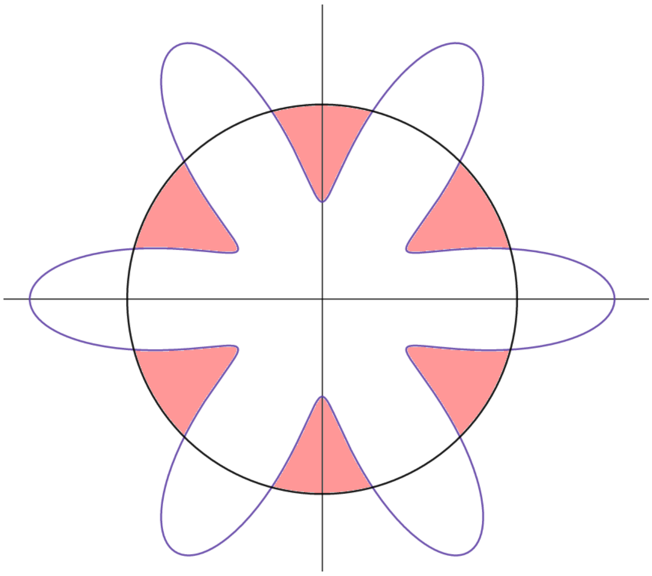
Hence find the roots of the equation

$$16x^6 - 24x^4 + 9x^2 - \frac{3}{4} = 0$$

giving your answers to three significant figures.

(5)

6.



Shown are the graphs of $r = 2 + \cos 6\theta$ and $r = 2$ for $0 \le \theta < 2\pi$. Find the pink shaded area.

(7)

7.

i)

Find the Maclaurin series expansion of $f(x) = e^{-x^2}$, up to the term in x^4, giving each term in its simplest form.

(4)

ii)

Use your series expansion to approximate the value of

$$\int_0^1 e^{-x^2}\,\mathrm{d}x$$

(2)

iii)

Use the integration function on your calculator to work out the percentage error in your approximation.

(1)

iv)

Explain why you would not use your series expansion to approximate the value of

$$\int_0^{10} e^{-x^2}\,\mathrm{d}x$$

(1)

8.

The point A is $(1, 5, 1)$ and the point B is $(-3, 10, -4)$.

The plane Π is defined by $\mathbf{r}.\begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} = 2$.

The point C is the reflection of B in Π. The point D is the reflection of A in Π.

Find the exact area of trapezium ABCD, giving your answer in the form $k\sqrt{35}$.

(10)

9.

The position of two particles, X and Y, along a line is modelled with the following equations. With respect to a fixed origin, the displacement of X is x centimetres and the displacement of Y is y centimetres, and t is the time in seconds.

$$\frac{dx}{dt} = 3x - 15y$$

$$\frac{dy}{dt} = 6x - 3y$$

i)

Show that

$$\frac{d^2x}{dt^2} + 81x = 0$$

(3)

ii)

Hence find the general solutions for x and y in terms of t.

(3)

iii)

Initially, X is at the origin with velocity 90cms^{-1}. Find the first time the two particles are at the same position, and find their exact displacement at this time.

(6)